E/C

Øvningsprøve

6.) $\int \frac{1}{x^2+4}\,dx = \frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx$

$= \frac{1}{4}\,\frac{\arctan\left(\frac{x}{2}\right)}{\frac{1}{2}} + C = \underline{\underline{\frac{1}{2}\arctan\left(\frac{x}{2}\right) + C}}$

7.) $\int \frac{\ln(x)}{x}\,dx = \int \ln(x)\cdot D(\ln(x))\,dx$

OBS: $\frac{1}{x} = D(\ln(x))$

$= \frac{(\ln(x))^2}{2} + C$

8.) $\int \frac{x}{x+1}\,dx = \int \frac{x+1-1}{x+1}\,dx =$

$\int \frac{x+1}{x+1} - \frac{1}{x+1}\,dx = \int 1 - \frac{1}{x+1}\,dx =$

$\underline{\underline{x - \ln(x+1) + C}}$

9.) $\int \underbrace{2x}_{f'(x)} \cdot \cos(\underbrace{x^2}_{f(x)})\,dx =$

$$\int D(x^2)\cdot\cos(x^2)\,dx = \sin(x^2) + C$$

10.) $$\int \frac{2x+1}{x^2-6x+8}\,dx = \int \frac{2x+1}{(x-4)(x-2)}\,dx$$

Partialbrikspjelling.

$$\frac{2x+1}{(x-4)(x-2)} = \frac{A}{(x-4)} + \frac{B}{(x-2)} \Rightarrow$$

$$2x+1 = A(x-2) + B(x-4) \Rightarrow$$

$$\begin{cases} A+B=2 \\ -2A-4B=1 \\ 2A+2B=4 \end{cases} \downarrow +$$

$$-2B = 5$$

$$B = -\frac{5}{2}$$

$$A = 2 + \frac{5}{2} = \frac{9}{2}$$

$$\frac{1}{2}\int \frac{9}{(x-4)} + \frac{-5}{x-2}\,dx =$$

$$\frac{1}{2}\cdot\left(9\ln(x-4) - 5\ln(x-2)\right) + C$$

7.) $\int \frac{\ln(x)}{x}\, dx$

U.S. $\ln(x) = t$

$x = e^t$

$\frac{dx}{dt} = e^t$

$dx = e^t \cdot dt$

$\int \frac{t}{e^t} \cdot e^t \cdot dt = \int t \cdot dt$

$= \frac{t^2}{2} + C = \frac{(\ln(x))^2}{2} + C$

7. $\int \frac{\ln(x)}{x}\,dx$

V.B $\ln(x) = t$

$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{dx}{x}$

$\int t\,dt = \frac{t^2}{2} + C$

$= \frac{(\ln(x))^2}{2} + C$